5ELEN018W - Tutorial 9 Exercises

1 Checkpoint

Based on the last tutorial session (Tutorial 8), none of the students finished all of the tutorial exercises.

Make sure that you finish them during this week (during the tutorial or on your own free time) as well as finishing all of the exercises since the beginning of the module.

If in doubt, you can show me your solution and get freedback.

Sample Solutions will be provided <u>ONLY</u> for some selected Exercises, not ALL!

2 The Inverted Pendulum Control Problem

The inverted pendulum is a classic problem in dynamics and control theory and is used as a benchmark for testing control strategies. It is implemented with the pivot point mounted on a cart that can move horizontally with the usage of a force F produced by a motor.

The objective is to control the pendulum so that it is in its upright position at all times ($\theta = 0$).

The problem is shown in Figure 1.

The equations of motion for the system are:

$$(M + m)\,\ddot{x} + m\ell \cos\theta\,\ddot{\theta} - m\ell\dot{\theta}^2 \sin\theta = F - b_c\dot{x}, \tag{1}$$

$$m\ell \cos\theta\,\ddot{x} + m\ell^2\,\ddot{\theta} - mg\ell \sin\theta = -b_p\dot{\theta}. \tag{2}$$

where

- Cart mass: $M = 1kg$
- Pole mass: $m = 0.1kg$
- Pole length: $l = 0.5m$
- Gravity: $g = 9.81m/s^2$
- Actuator (force) limits (typical): $F_{min} = -10N, F_{max} = +10N$
- Viscous friction on cart: $b_c = 0.10N \cdot s/m$

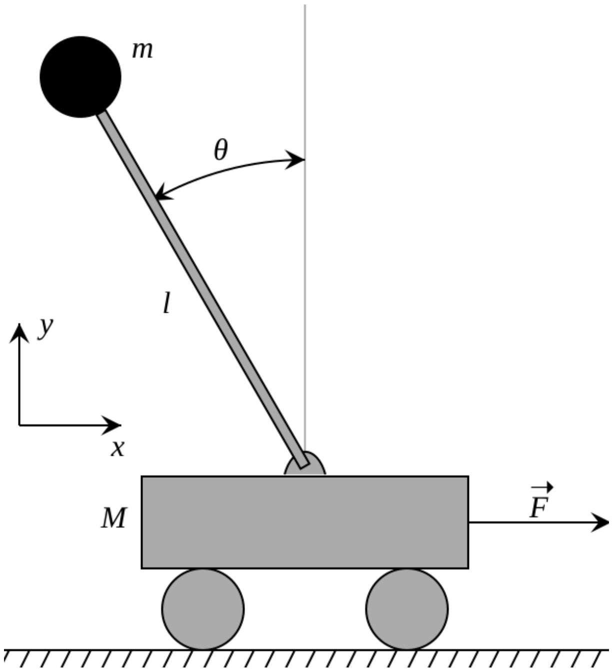

Figure 1: The inverted pendulum control problem.

- Viscous friction at the pendulum pivot: $b_p = 0.01N \cdot m \cdot s/rad$

1. Model the dynamic system using Simulink.
2. Attempt to control the system use a PID controller. Can you find a set of parameters which solves the balancing problem?
3. Plot the Bode diagrams for the system based on your PID controller. What can you say about the stability of the system?