# 5ELEN018W - Tutorial 8 Exercises: Forward Kinematics

## 1 Combined Transformations

A point $P$ with coordinates $[7, 3, 1]^T$ is attached to a frame $F$ which is subject to the following 3 successive transformations relative to the reference frame $F_{xyz}$:

1. Rotation of $90^o$ about the $z$-axis.
2. Followed by a rotation of $90^o$ about the $y$-axis.
3. Followed by a translation of $[4,\ -3,\ 7]$.

Find manually (using a calculator, you could use one of the software implemented calculators) the coordinates of the point relative to the reference frame after all the transformations have taken place.

Verify your calculations by using Python and doing the same calculations using matrices.

**Hints:** See the lecture 4 slides. The order of the matrices written (calculated) is the opposite of the order of transformations performed. The result should calculate to $[5,\ 4,\ 10,\ 1]$.

## 2 DH Matrix Derivation

Consider the following DH table:

| **Joint** | $\boldsymbol{\theta}$ | $\boldsymbol{d}$ | $\boldsymbol{r}$ | $\boldsymbol{\alpha}$ |
|---|---|---|---|---|
| 1 | $\theta$ | $d$ | $r$ | $\alpha$ |

1. Use Python to derive the DH matrix by applying the 4 basic transformations shown on the left hand side of Equation (6) of the last lecture.

   Use `SymPy` to create the corresponding symbols before creating the 4 basic matrix transformations.

2. Check that your calculated DH matrix matches with the one shown on the right hand side of Equation (7) of the last lecture slides.

# 3 DH Matrix

Consider a 2-joint robot with the following DH table:

| **Joint** | $\boldsymbol{\theta}$ | $\boldsymbol{d}$ | $\boldsymbol{r}$ | $\boldsymbol{\alpha}$ |
|---|---|---|---|---|
| 1 | $\theta_1$ | $d_1$ | $r_1$ | $\alpha_1$ |
| 2 | $\theta_2$ | $d_2$ | $r_2$ | $\alpha_2$ |

Determine the position of the robot's end-effector. Use symbolic Python (`SymPy`) for the calculations.

# 4 3-DOF Robot - Forward Kinematics Problem

Consider the 3-DOF robot shown in Figure 1. Its DH table is given by:

| **Joint** | $\boldsymbol{\theta}$ | $\boldsymbol{d}$ | $\boldsymbol{r}$ | $\boldsymbol{\alpha}$ |
|---|---|---|---|---|
| 1 | $\theta_1$ | 0 | $r_1$ | 0 |
| 2 | $90 + \theta_2$ | 0 | 0 | 90 |
| 3 | $\theta_3$ | $d_3$ | 0 | 0 |

Determine the position of the robot's end-effector. Use symbolic Python for the calculations.

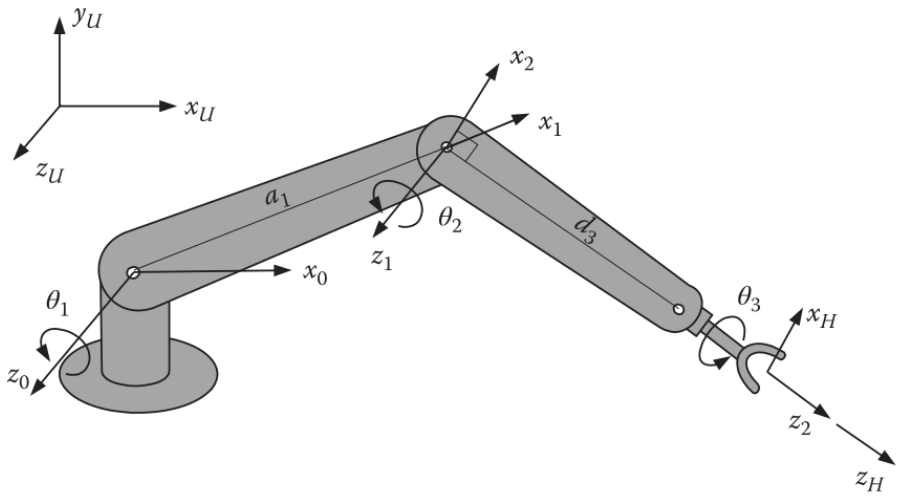



Figure 1: The 3-DOF robot of Exercise 4.

# 5 Python Programming

Write a Python function (see Week 1) which accepts 4 arguments: $\theta, d, r, \alpha$ and returns the DH matrix corresponding to these parameters for a single robot joint.

# 6 The Puma 560 Industrial Robot

The Unimation PUMA 560 is an industrial manipulator robot with 6 degrees of freedom and all rotational joints (i.e. it is a 6R mechanism).

It is shown in Figures 2 and 3.

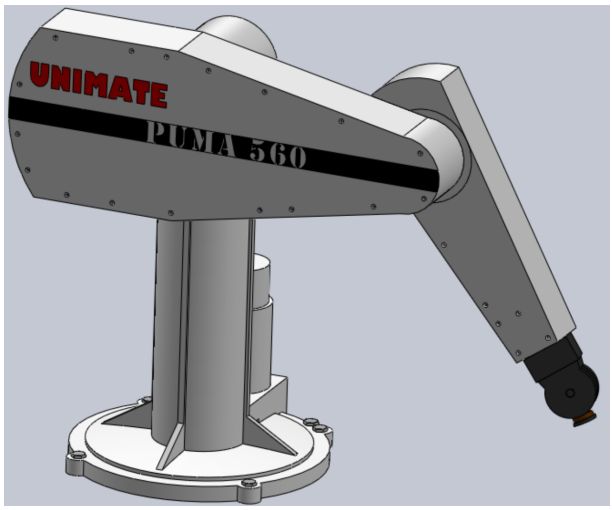



Figure 2: The Puma 560 Industrial Robot.

The DH table for its joints is:

| **Joint** | $\boldsymbol{\theta}$ | $\boldsymbol{r}$ | $\boldsymbol{d}$ | $\boldsymbol{\alpha}$ |
|---|---|---|---|---|
| 1 | $\theta_1$ | 0 | 0 | 0 |
| 2 | $\theta_2$ | 0 | 0 | $-90^o$ |
| 3 | $\theta_3$ | $r_2$ | $d_3$ | 0 |
| 4 | $\theta_4$ | $r3$ | $d_4$ | $-90^o$ |
| 5 | $\theta_5$ | 0 | 0 | $90^o$ |
| 6 | $\theta_6$ | 0 | 0 | $-90^o$ |

Use the implemented Python function of the previous Exercise 5, calculate the position of the Puma 560 tool (end-effector).

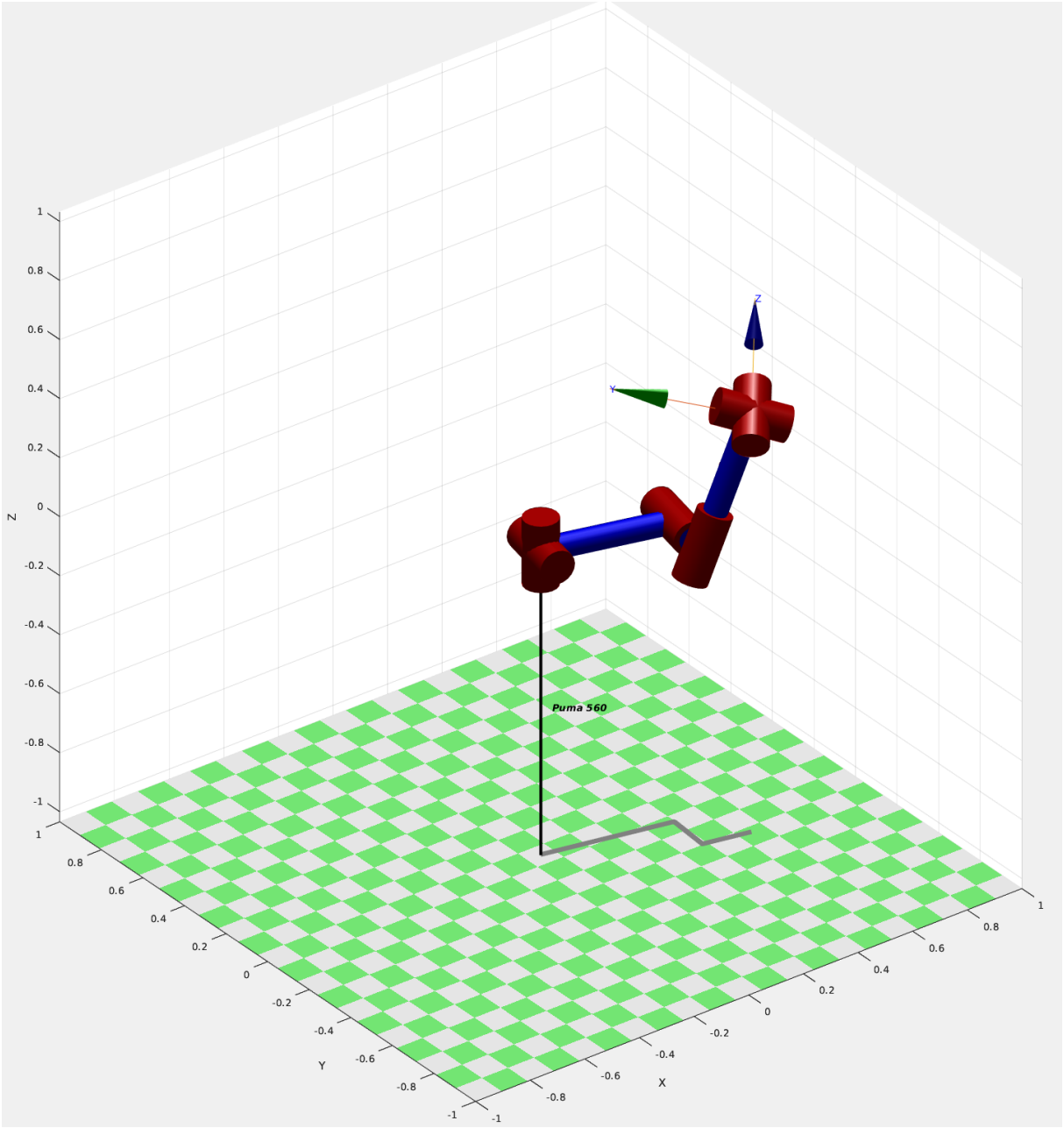



Figure 3: The Puma 560 Robot model (Peter Corke's toolbox).