5ELEN018W - Robotic Principles
Lecture 8: Control - Part 3

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Analysis of Control Systems - Performance of Controllers

- steady-state error
- transient response
- overshooting
- settling time
- stability

Stability

Stability is very important.

▶ A stable system should have a bounded output if the corresponding input is bounded.

Stability in Systems - Example

The Tacoma Narrows bridge in Washington was opened to traffic on 1st of July 1940.

- ▶ The bridge was oscillating every time the wind blew.

Stability in Systems - Example (cont'd)

▶ After 4 months (November 7th 1940) the bridge collapsed.

https://www.youtube.com/watch?v=y0xohjV7Avo

Time Response Analysis

Plots of the plant's output vs the desired behaviour can be used for the analysis of controllers.
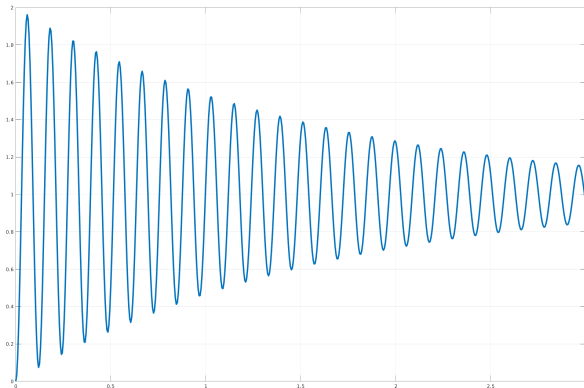
▶ **Step response:** the control system performance specifications are often given in terms of the *unit step response*.

Example of a Unit Step Response

System having the following transfer function:

$$\frac{5400}{2s^2 + 2.5s + 5402} \tag{1}$$

Example of a Unit Step Response (cont'd)

Matlab code:

```
num = [5400];
den = [2 2.5 5402];
sys = tf(num, den)
t = [0:0.005:3];
[y,t] = step(sys,t);
plot(t,y,'LineWidth',3),grid
xlabel('time(s)')
ylabel('System output')
```

Frequency Response

The frequency response of a system is defined as the steady-state response of the system to a sinusoidal input signal. The output for a linear system is also sinusoidal with the same frequency.

- ▶ Although the output response has the same frequency as the input, they have different amplitude and phase angle.
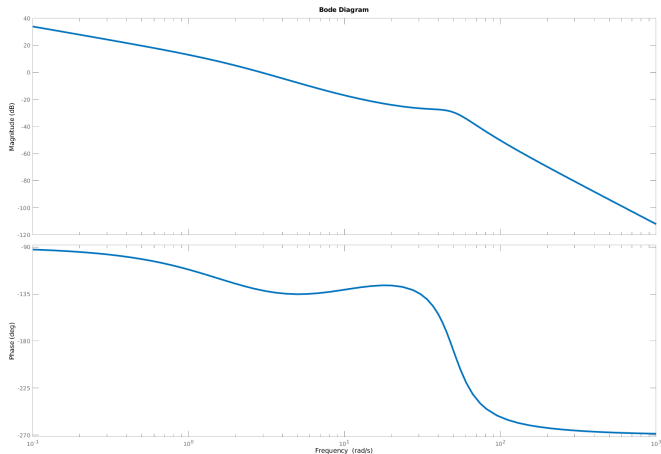
Bode Plots

Hendrik Wade Bode an engineer working in Bell Labs in the 1930s came up with Bode plots, which can be used to determine the stability of systems.

The plot consists of 2 diagrams:

▶ The magnitude plot (magnitude in decibels), frequency in rad/s. (frequency values are logarithmic, while the magnitude $|H|$ values are in decibels, i.e. $20 \log_{10} |H|$.

▶ The phase plot (phase in degrees), frequency in rad/s. Frequency values are in the logarithmic scale but the phase values are in the linear scale.

Example of a Bode Plot


```
sys=tf(num, den);   % previous system example
bode(sys)
```

Gain Margin and Phase Margin in Bode Plots

The gain margin and the phase margin can be used to determine the stability of the system.

To calculate the **gain margin**:

1. Find the frequency corresponding to a phase of -180° from the phase plot.

2. For that frequency find the corresponding magnitude M_1 from the magnitude plot.

3. The **gain margin** is the difference between the magnitude of $0dB$ and M_1: $0 - M_1$.

Gain Margin and Phase Margin in Bode Plots (cont'd)

To calculate the **phase margin**:

1. Find the frequency corresponding to a magnitude of $0dB$ from the magnitude plot.

2. For that frequency find the corresponding phase P_1 from the phase plot.

3. The **phase margin** is the difference between P_1 and -180°: $P_1 - (-180) = P_1 + 180$.

How to determine Stability from the Bode Plots

Stability can be determined by calculating either the gain margin or the phase margin:

- ▶ A positive gain margin indicates a stable system, while a negative gain margin indicates an unstable system. The larger the positive gain margin, the system has more "margin" before going to instability.

- ▶ A positive phase margin indicates a stable system, while a negative phase margin indicates an unstable system. The larger the positive margin the more stable is the system and a value of at least 45 degrees is desirable.

Example 2 of a Bode Plot
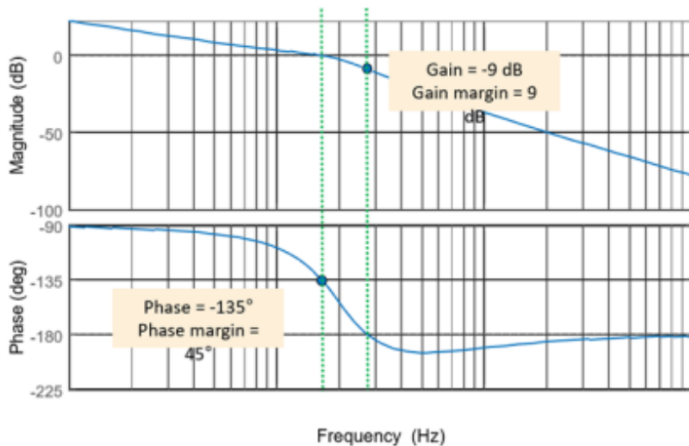
Example 3 of a Bode Plot
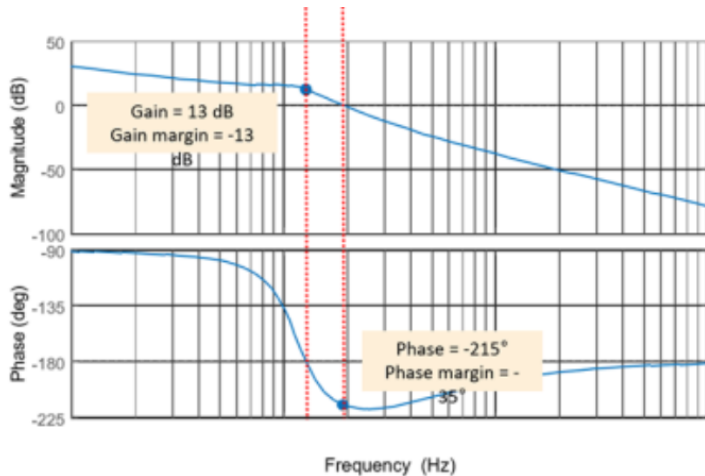
Bode Plots in Simulink
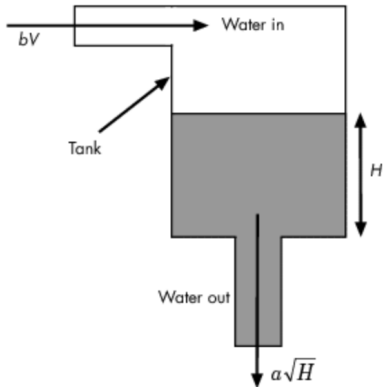
Problem: Watertank

Figure 1: The watertank dynamic system.

Bode Plots in Simulink (cont'd)

$$\frac{d}{dt} Vol = A \frac{dH}{dt} = bV - \alpha\sqrt{H}$$

- ▶ H is the height of the water in the tank
- ▶ Vol is the volume of the water in the tank
- ▶ $b = 5$ is a constant related to the flow rate into the tank
- ▶ $A = 20$ relates to the area of the tank
- ▶ $\alpha = 2$ is a constant related to the flow rate out of the tank

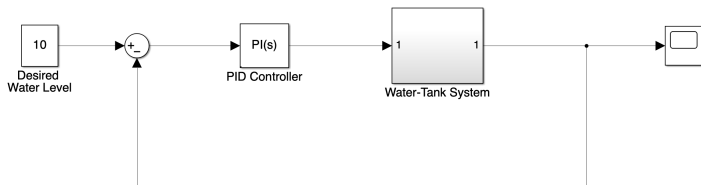# Bode Plots in Simulink (cont'd)

Figure 2: The watertank dynamic system in Matlab (built-in example up to version 2022).
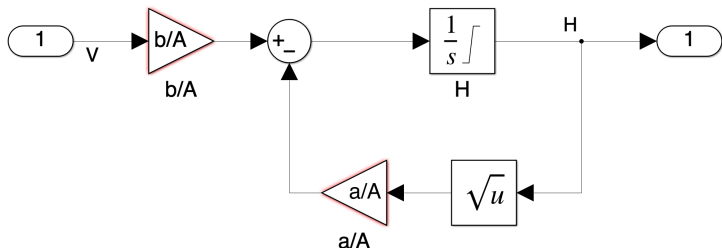
Bode Plots in Simulink (cont'd)

Figure 3: The watertank dynamic system in Matlab (built-in example up to version 2022).

Bode Plots in Simulink (cont'd)

From the Simulink Library Browser:

1. Choose Simulink Control Design
2. Choose Linear Analysis Plots
3. Choose and add a Bode Plot block
4. Open the Bode Plot block and in Linearisation add 2 signals:
   - $\rightarrow$ The output of the PID controller as input-perturbation
   - $\rightarrow$ The output of the plant as open-loop output
5. Run to display the Bode plot.

Bode Plots in Simulink (cont'd)