5ELEN018W - Robotic Principles
Lecture 8: Control - Part 3

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The Laplace Transform

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$$ms^2 + bs + k = F \tag{2}$$

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- These are used to make easier the modelling and analysis of dynamic systems.

Feedback (Closed-Loop) Controllers Transfer Function

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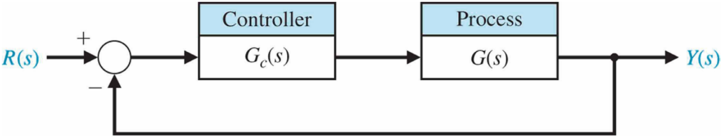

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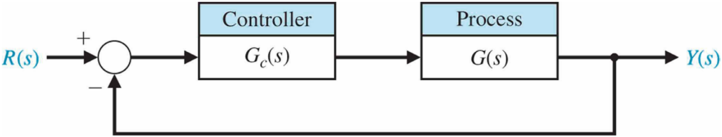

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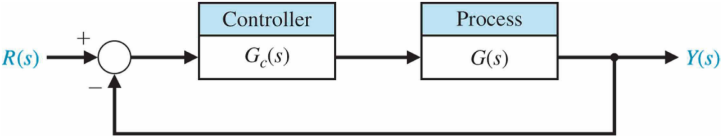

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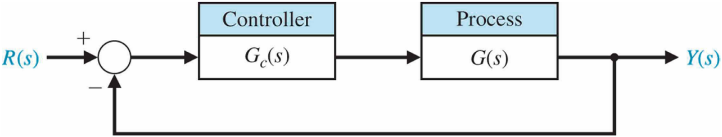

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- Stability is guaranteed when $G(s)G_c(s) < -1$.

PID Control for the Robot Arm Surgeon

$$m\ddot{x} + b\dot{x} + kx = f \tag{5}$$

The desired position is 1, starting at position $x = 0$. For all the simulations, the following parameters were used:
$m = 1, b = 6, k = 9.86960$.

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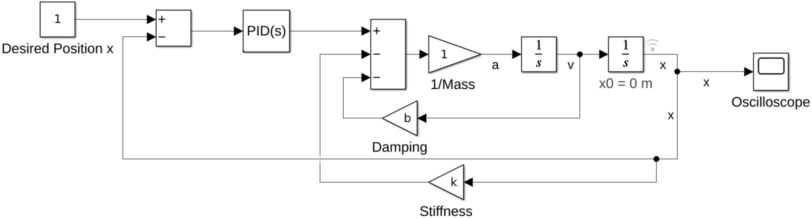

P-Controller

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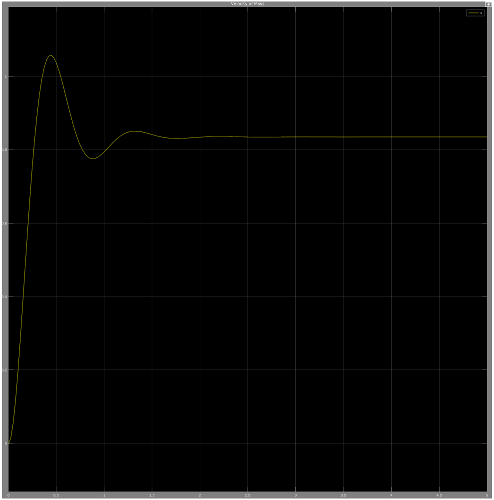

P-Controller (cont'd)

- large steady-steady error

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- large overshoot

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- large overshoot
- large settling time

PD-Controller

$K_p = 50, K_d = 2.5$

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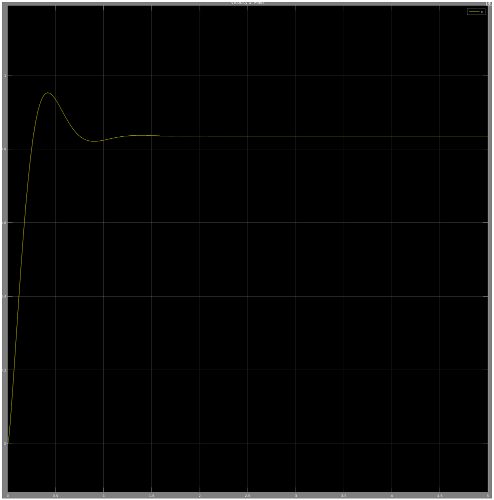

PD-Controller (cont'd)

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The *D*-component has reduced:

- the overshoot

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PD-Controller (cont'd)

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- the overshoot
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Still large steady-state error.

PI-Controller

$K_p = 50, K_i = 40$

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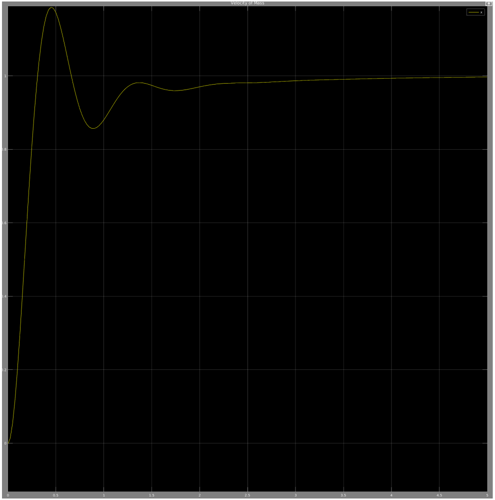


PI-Controller

PI-Controller

- 0 steady-steady error

PI-Controller

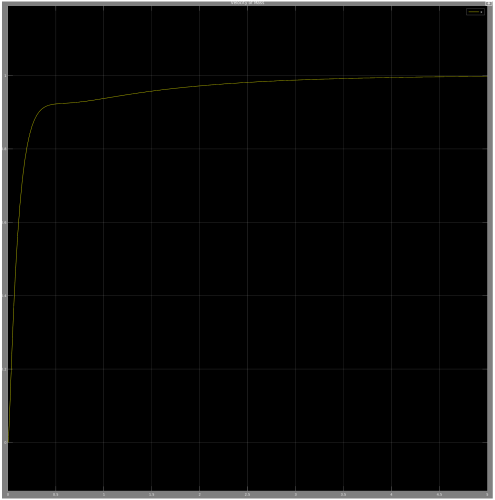
- 0 steady-steady error
- still large overshoot

PID-Controller

$K_p = 50, K_i = 40, K_d = 8$

PID-Controller

$K_p = 50, K_i = 40, K_d = 8$



PID-Controller (cont'd)

- no steady steady error
- no overshoot
- faster rise time

Implementing a Dynamic System and a Controller Programmatically (not Simulink)

Car cruise control can be found in most of the modern cars. The cruise control system keeps the car at a constant speed despite any external disturbances, such as the road surface and incline.

- The car's mass m is controlled by a force u. To simplify the situation, we assume that the force u that our controller applies is not affected by other parameters such as tires, etc.

The equation of the system is described by the following:

$$m\dot{v} + bv = u \tag{6}$$

where v is the speed of the car, u is the control action and b is the damping coefficient due to friction.

The Car Cruise Control System

$$m\dot{v} + bv = u \tag{7}$$

- $m = 1000, b = 50$.
- The desired (reference) speed is $v_{ref} = 10$.
- $t = 10.0$ is the total simulation time
- The parameters of the PID controller are:
  $K_p = 800, K_i = 0, K_d = 40$ (i.e. this is a PD controller).

Implement in Java a PID controller to bring the system to the desired speed.

The Car Cruise Control System (cont'd)

```
import java.io.*;

class CruiseControl {
    static double v = 0;  // current speed initialised to 0
    static double previous_v = 0;  // the speed at the previous time step
    static double dt = 0.001;  // time step for the simulation


    /* Implementnts the dynamic system (plant) - system input is action u
       and the method returns the output of the plant */
    static double plant(double action_u) {
        // m \dot{v} + b v  = u
        // m=1000, b=50, u = 500
        double m = 1000.0;
        int b = 50;

        double v_dot = (action_u - b*v)/m;

        double new_speed = v + v_dot*dt;
        return new_speed;
    }
```

The Car Cruise Control System (cont'd)

```
public static void main(String[] args) {
    PrintWriter pw = null;

    try {
        pw = new PrintWriter("myfile.txt");
    }
    catch(Exception ex) {
        ex.printStackTrace();
    }

    double start_time = 0;
    double end_time = 10.0;

    double current_time = start_time;

    double v_ref = 10;  // the desired speed

    int K_p = 800;  // proportional gain
    int K_i = 0;    // integral gain
    int K_d = 40;   // derivative gain

    double previous_error = 0;
    double integral = 0;
```

The Car Cruise Control System (cont'd)

```
/* simulate the system operation from the beginning till
   the end of the simulation */
while (current_time <= end_time) {
    double error = v_ref - v;

    // I(ntegral) component of the PID controller
    integral = integral + error*dt;

    // D(erivative) component of the PID controller
    double deriv = (error - previous_error)/dt;

    // the output (action) of the PID controller
    double action = K_p*error + K_i*integral + K_d*deriv;

    // remember the last error when the previous action
    // was applied to the plant
    previous_error = error;

    // apply the new action to the plant to calculate
    // the new (current) speed
    v = plant(action);
```

The Car Cruise Control System (cont'd)

```
            System.out.println("Time: " + current_time + " Action: " +
                                    action + ", Speed=" + v);
            pw.println(v + " " + current_time);

            // advance the time
            current_time += dt;
        }

        pw.close();
    }
}
```