5ELEN018W - Robotic Principles
Lecture 4: Kinematics

Dr Dimitris C. Dracopoulos

More On Transformations

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$$\boldsymbol{Transform}_{xyz} = \boldsymbol{R}_y \cdot \boldsymbol{T}_x \cdot \boldsymbol{R}_z$$

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$$\boldsymbol{p}_{xyz} = \boldsymbol{Transf}_{x'y'z'} \times \boldsymbol{p}_{x'y'z'} \quad (2)$$

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e.g. for a rotation about the z' axis followed by a translation about the x' axis, followed by a rotation about the y' axis:

$$\boldsymbol{Transform}_{x'y'z'} = \boldsymbol{R}_{z'} \cdot \boldsymbol{T}_{x'} \cdot \boldsymbol{R}_{y'}$$

Forward Kinematics vs Inverse Kinematics

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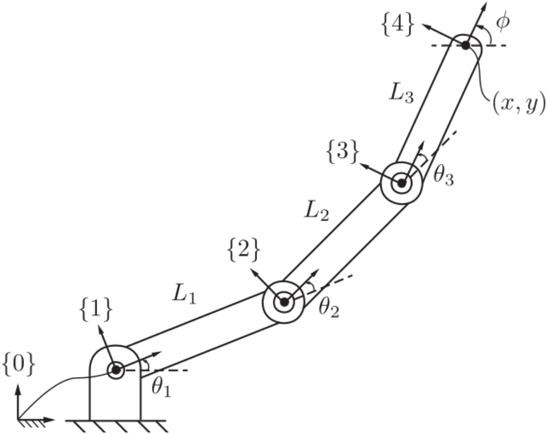
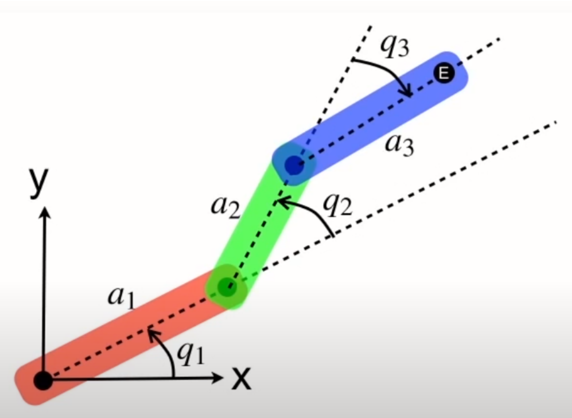
- *Forward Kinematics*: the calculation of the position and orientation of a robot's end-effector from its joint coordinates θ.
- *Inverse Kinematics*: given a position and orientation of a robot's end-effector, calculate the angles θ of the joints.

Forward Kinematics



Forward Kinematics



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The position and orientation of all links.

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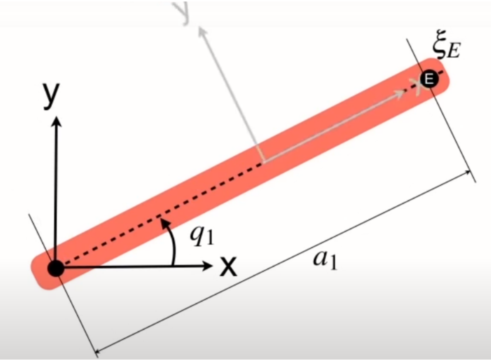
$$HomogeneousMatrix = Transf_1 * Transf2 * Transf3 * \ldots * Transf_n \quad (3)$$

where n is the number of links (assuming that each of these matrices is the <u>total</u> transformation for each link).

Example of a 1-joint Robot Arm



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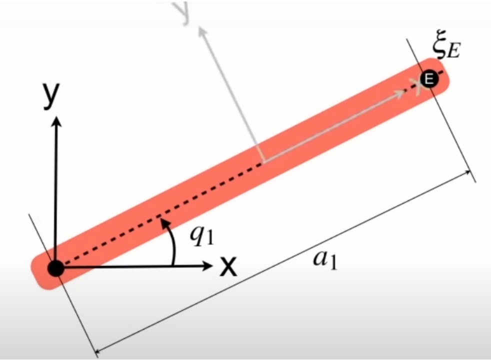

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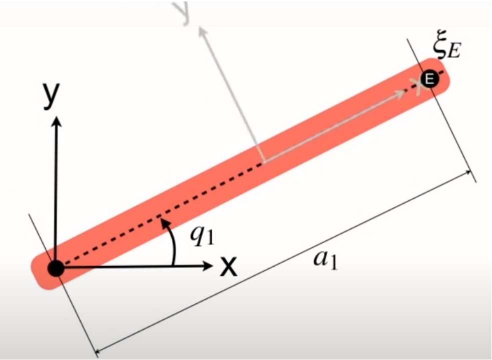

Rotation by angle q_1 and then translation by a_1.
The homogeneous transformation describing the overall result can be calculated using the following:

$$EndEffector = Rot(q_1) \cdot T_x(a_1) \tag{4}$$

Pose of the End-Effector - 1-Joint 2D Robot Arm (cont'd)

In Matlab Robotics Toolbox:

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>> e.fkine(pi/2)
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>> a1 = 1

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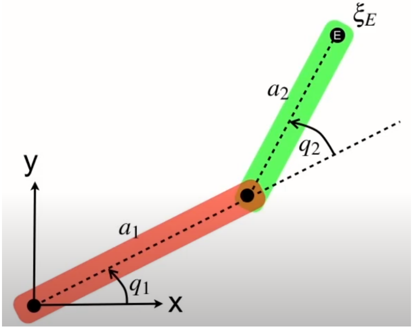
Example of a 2-joint Planar Robot Arm



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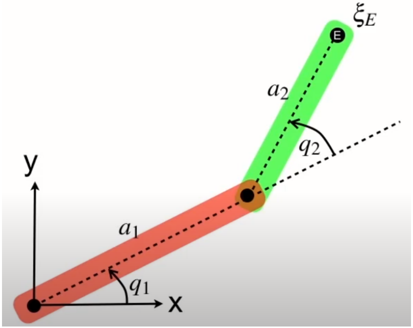

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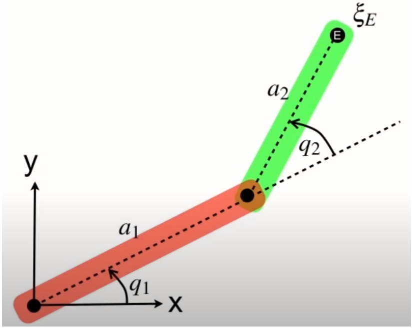

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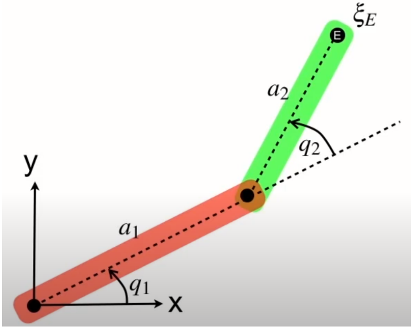

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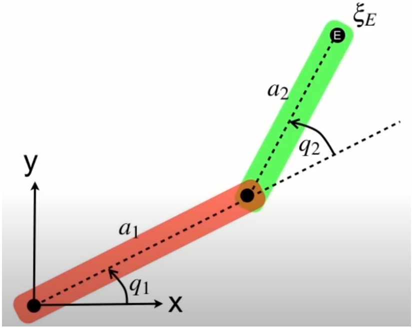

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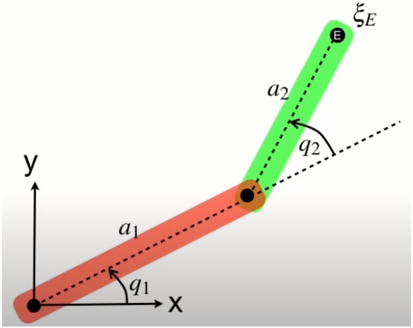

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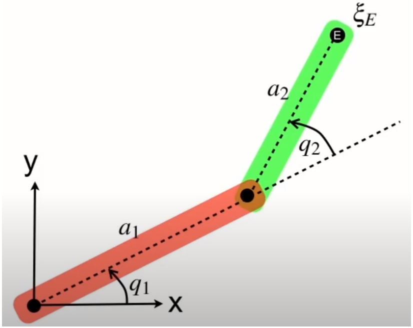

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The homogeneous transformation describing the overall result can be calculated using the following:

$$EndEffector = Rot(q_1) \cdot T_x(a_1) \cdot Rot(q_2) \cdot T_x(a_2) \quad (5)$$

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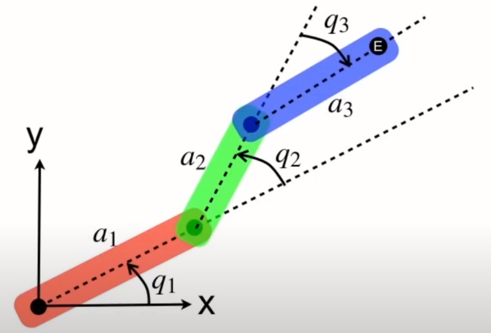

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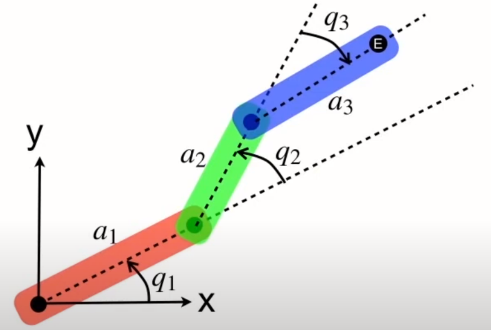

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using simple transformations in Mathematics which correspond to real operations in Physics!

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$\longrightarrow$ 6 parameters - 2 constraints means 4 parameters are needed.

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Each homogeneous transformation A_i is represented as the product of 4 basic transformations:

$$A_i = Rot_{z,\theta_i} \cdot Trans_{z,d_i} \cdot Trans_{x,r_i} \cdot Rot_{x,\alpha_i} \tag{6}$$

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$$A_i = Rot_{z,\theta_i} \cdot Trans_{z,d_i} \cdot Trans_{x,r_i} \cdot Rot_{x,\alpha_i} =$$

$$\begin{bmatrix} \cos(\theta_i) & -\sin(\theta_i) & 0 & 0 \\ \sin(\theta_i) & \cos(\theta_i) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\times \begin{bmatrix} 1 & 0 & 0 & r_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\alpha_i) & -\sin(\alpha_i) & 0 \\ 0 & \sin(\alpha_i) & \cos(\alpha_i) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

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- For *prismatic joints*: Only d changes, all the other 3 parameters are fixed according to the robot mechanism.

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To calculate, apply Equation (7).

Finding the Pose of the End-Effector relative to the Base Frame

Assume that $A_1, A_2, A_3 \ldots A_n$ are the DH matrices of all the robot joints $1, 2, 3, \ldots n$.

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Then the calculation requires the multiplication of all the matrices:

$$Pose_{end\_effector} = A_1 \cdot A_2 \cdot A_3 \ldots A_n \qquad (8)$$

Example: Calculation of the Pose of the End-Effector

The following DH matrices correspond to the joints of a robot, from robot base to end-effector. Find the pose of the end-effector relative to the robot base.

$$A_1 = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} -1 & 0 & 0 & -2 \\ 0 & -0 & 1 & 0 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Example: Calculation of the Pose of the End-Effector (cont'd)

Example: Calculation of the Pose of the End-Effector (cont'd)

Simply calculate $A_1 * A_2 * A_3$.